Generalized Net Representations of the Causal Structure of a Queuing System

Velin Andonov
Institute of Mathematics
and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., bl. 8,
Sofia 1113, Bulgaria
Email: velin_andonov@math.bas.bg

Stoyan Poryazov
Institute of Mathematics
and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., bl. 8,
Sofia 1113, Bulgaria
Email: stoyan@math.bas.bg

Emiliya Saranova
Institute of Mathematics
and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., bl. 8,
Sofia 1113, Bulgaria
Email: emiliya@math.bas.bg

***Abstract*—Queuing systems are an important part of virtually all service networks. The problem of determining how the Quality of Service (QoS) of a queuing system depends on the qualities of the buffer and the server is not well studied. Two causal decompositions of the services in a queuing system are considered which use the elements of Service Systems Theory. For each of them, a Generalized Net (GN) representation is proposed. The GN models can be used in the study of the QoS composition in overall telecommunication networks.**

***Keywords*—generalized nets; queuing systems; quality of service composition**

I. Introduction

In recent years, the importance of the Quality of Service (QoS) in information service networks has experienced a growth. One of the reasons for this is that QoS has become a commodity (see [1]). Various definitions for QoS and Quality of Experience (QoE) exist in the literature. The most widely accepted are the definitions in the standardization documents of the International Telecommunication Union (ITU, [2]). ITU-T Recommendation E.800 from 2008 (see [3]) defines QoS as "the totality of characteristics of a telecommunication service that bear on its ability to satisfy stated and implied needs of the user of the service". The key quality indicators, in telecommunications, are defined in the ITU recommendations. They are of different types. The types of QoS indicators values may vary:

- Qualitative - quantitative;
- Discrete - continuous;
- They have different ranges;
- Indicators may be a decreasing (e.g. blocking probability) or increasing (e.g. call efficiency ratio) function of the corresponding QoS.

In the most detailed existing model of an overall telecommunication network, including users' behavior, described in [4], the QoS is determined by the probability of blocked ringing (Pbr) and the probability of blocked switching (Pbs). Other indicators for QoS in overall telecommunication networks are proposed in [5].

The classical model of overall telecommunication network is extended in [6] with the inclusion of a queuing system, including a buffer and a server, in the switching stage of the telecommunication network. The mean wating time of the requests in the buffer of the queuing system and the probability of blocked switching are important indicators for the QoS.

The problem of predicting the QoS of an overall network as an aggregation of the qualities of composed services is an important question in the service design and the maintenance of the network. In the present paper, we focus on the representation of the composition of services and the quality aggregation of these services in a queuing system as a part of an overall telecommunication network.

The conceptual modeling approach used here is based on the apparatus of the Generalized Nets (GNs) theory [8]. GNs are used in the modeling of overall telecommunication systems in [9]. More recently, in the papers [10], [11] for the first time GNs models of queuing systems with different service disciplines are proposed. The GNs approach to the conceptual modeling of queuing systems is compared to the Service Systems Theory approach in [12].

The paper consists of the following sections. Section II presents the notions of base and comprise virtual devices, causal structure of the devices, parameters and a system for notation. In Section III, a general causal structure of a queuing system is proposed and a GN representation of it is proposed. In Section IV, a detailed representation of the causal structure of a queuing system is proposed and a GN model is constructed based on it. In Section V, some conclusions are made.

II. Terms and definitions

The building blocks of the conceptual models of overall telecommunication networks in general, and a queuing system in the context of a telecommunication system, are the base virtual devices. A general graphical representation of a base virtual device is shown in Fig. 2.

Every base virtual device x has six parameters important for the analytical modeling:

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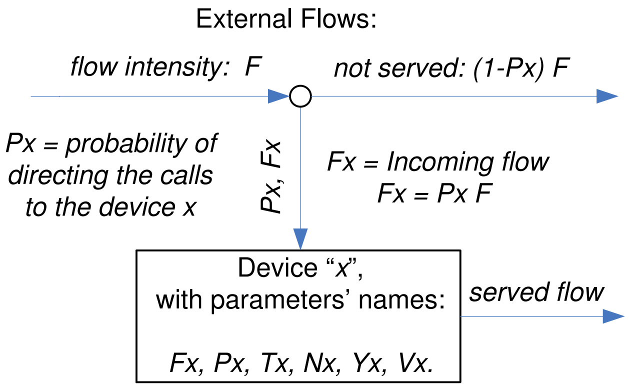

Fig. 1. Graphical representation of a base virtual device.

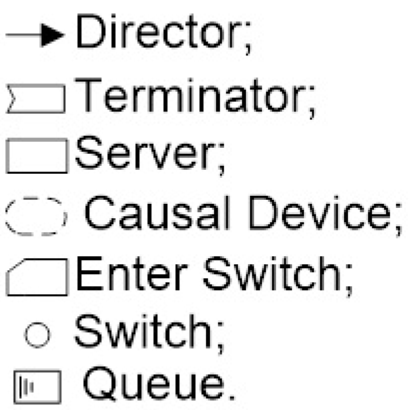

Fig. 2. Base virtual device types and their graphical representations.

- Fx - intensity or incoming rate (frequency) of the flow of requests (i.e. the number of requests per time unit) to device x;
- Px - probability of directing the requests towards the device x;
- Tx - service time (duration of servicing of a request) in device x;
- Yx - traffic intensity [Erlang];
- Vx - traffic volume [Erlang - time unit];
- Nx - number of lines (service resources, positions, capacity) of device x.

Different types of base virtual devices are used in the models. The ones used in the paper together with their graphical representations are shown in Fig. 2

The devices of each type have specific functions (see [4]):

- Director – points to the next device to which the request is transferred without delay;
- Terminator – eliminates each request which enters it;
- Server – models traffic and time characteristics of the model, the delay (service time, holding time) of the requests;
- Causal Device – represents carried and parasitic services;
- Enter Switch – deflects a part of the requests before a switch;
- Switch – selects one of its possible exits for each request entered, thus determining the next device where this request shall go to.
- Queue – buffer device of the queuing system.

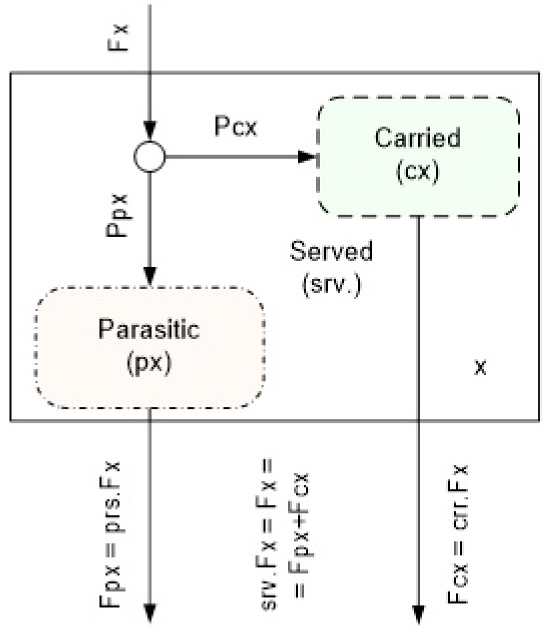

Fig. 3. Graphical and textual presentations of causal sub-devices ’Parasitic’ (’px’) and ’Carried’ (’cx’) of the virtual device ’x’, and some of their parameters. Ppx and Pcx are the corresponding probabilities of directing of the requests’ flows; Fpx and Fcx are flow intensities; Fx is the intensity of the input flow; $srv.Fx$ - of the output. Due to the stationarity assumption: $Fx = srv.Fx$.

Special qualifiers are used to characterize the flow intensity (F) and the traffic $(Y$, see [4]). For example, $inc.Fs$ is the flow intensity of the incoming requests to the **s** device (the switching system); $ofr.Ys$ is the offered traffic to the switching system, etc.

New qualifiers are introduced here to represent the causal structure of a queuing system. The qualifiers used in the paper are:

- par. – comes from “parasitic”. Used to denote the parasitic flow and traffic;
- srv. – comes from “served”. Used to denote the served flow of requests;
- nsr. – comes from “not served”. It denotes a fictive traffic;
- blc. – comes from “blocked”. Denotes the blocked traffic;
- gen. – comes from “genuine”. Denotes a real service with non-zero service time;
- zer. – comes from “zero”. Denotes a service with zero service time.

In the conceptual modeling of the causal structure of virtual devices the so called “causal sub-devices” are used (see Fig. 3).

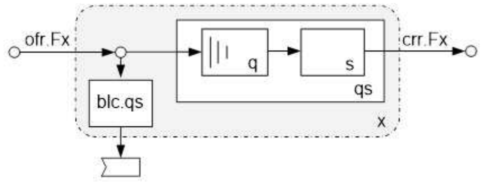

Fig. 4. Graphical representation of the causal structure of a queuing system in Service Systems Theory.

III. Simple causal structure of a queuing system

A Service Systems Theory model of the causal structure of a queuing system, including buffer and server, is shown in Fig.4

The queuing system is represented by the **x** comprise virtual device. It consists of a base virtual device **blc.qs** which models the service of the blocked requests due to lack of resources, and comprise virtual device **qs**. The **qs** device comprises of a base virtual device of type buffer (**q**) and a server device (**s**). The offered flow of requests to the queuing system is denoted by $ofr.Fx$. When the **qs** device has reached its capacity, i.e., both the **q** and **s** device have reached their capacities, the requests enter the blocked branch ($blc.qs$) and leave the model through the terminator device after it. If the **qs** device has not reached its capacity, the requests enter the buffer and from there they are sent to the server directly without waiting (if the device **s** has not reached its capacity) or wait until there are free service lines in the **s** device. The carried flow of requests by the queuing system is $crr.Fx$.

The GNs are used as an alternative approach to the conceptual modeling of queuing systems (see [10], [11], [12]). GN representations of basic elements of Service Systems Theory are proposed in [13] which allow every model in Service Systems Theory to be easily represented by a GN. In these papers, the problem of representation of the comprise virtual devices is not studied. This problem is especially evident in the causal structure representation in Fig.4 in which we have devices comprised on two levels. We propose a GN representation of the simple causal structure (Fig.4) in which the comprise devices are included in the gaphical representation. The GN model is shown in Fig.5.

The GN model consists of 5 transitions and $13 + Nq$ places where Nq is the capacity of the buffer. The labels of those places which represent virtual devices are in the form l_y where "y" is the name of the corresponding virtual device. The transitions represent functions of the corresponding virtual devices as follows:

- Z_1 represents the function of the Switch before the **qs** device from Fig. 4.
- Z_2 represents the function of the Director pointing to the **q** device from Fig. 4.
- Z_3 represents the function of the Director between the **q** device and the **s** device from Fig. 4.
- Z_4 represents the function of the comprise virtual device **qs** from Fig. 4.

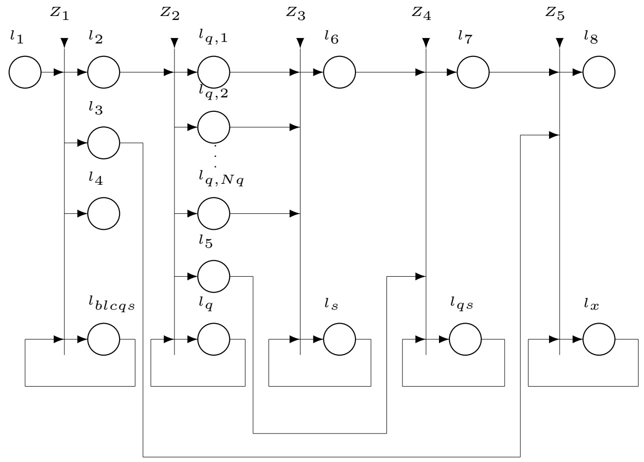

Fig. 5. GN model of the simple causal structure representation of a queuing system.

- Z_5 represents the function of the comprise virtual device **x** from Fig. 4.

The places of the GN correspond to virtual devices in the following way:

- l_{blcqs} represents the **blc.qs** device in Fig. 4.
- $l_{q,1}, ..., l_{q,Nq}$, l_q represent the waiting places of the buffer device **q**.
- l_s represents the server device **s**.
- l_{qs} represents the comprise virtual device **qs**.
- l_x represents the comprise virtual device **x**.

Six types of tokens are used in the model:

- Tokens of type α represent the requests. In the initial time moment of the GN functioning, token α stays in place l_1 with initial characteristic "formula for generating the offered flow of requests to the queuing system".
- Token of type β stays in place l_{blcqs} in the initial time moment with initial characteristic "initial values of $Y_{blc.qs}, P_{blc.qs}, F_{blc.qs}, T_{blc.qs}, N_{blc.qs}$ ". It is used to accumulate data about the **blc.qs** device.
- Token of type γ stays in place l_q in the initial time moment with initial characteristic "initial values of $Y_q, P_q, F_q,\ T_q, N_q$". It is used to accumulate data about the **q** device. The discipline of service of the requests can be specified in this initial characteristic. Here, we consider only FIFO (First-In, First-Out) discipline of service of the requests.
- Token of type δ stays in place l_s in the initial time moment with initial characteristic "initial values of Y_s, P_s, F_s, T_s, N_s". It is used to accumulate data about the **s** device.
- Token of type ϵ stays in place l_{qs} in the initial time moment with initial characteristic "initial values of $Y_{qs}, P_{qs}, F_{qs},\ T_{qs}, N_{qs}$". It is used to accumulate data about the **s** device.

- Token of type ζ stays in place l_x in the initial time moment with initial characteristic "initial values of Y_x, P_x, F_x, T_x, N_x". It is used to accumulate data about the **x** device.

The formal description of the GN transitions follows below.

$$Z_1 = \langle \{l_1, l_{blcqs}\}, \{l_2, l_3, l_4, l_{blcqs}\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|cccc} & l_2 & l_3 & l_4 & l_{blcqs} \\ \hline l_1 & W_{1,2} & true & false & W_{1,blcqs} \\ l_{blcqs} & false & false & true & true \end{array}$$

and

- $W_{1,2}$ ="$Yqs < Nqs$";
- $W_{1,blcqs} = \neg W_{1,2}$.

When the truth value of the predicate $W_{1,2}$ is "true" the α token in place l_1 splits into two identical tokens – the same α token which enters place l_2 and token α_1 which enters place l_3 with characteristic "*ofr.Fx*". The β token in place l_{blcqs} splits into two tokens one of which remains in place l_{blcqs} and obtains the characteristic "*current values of* $Y_{blc.qs}, P_{blc.qs}, F_{blc.qs}, T_{blc.qs}, N_{blc.qs}$ ", while the other enters place l_4 without obtaining new characteristic.

$$Z_2 = \langle \{l_2, l_q\}, \{l_{q,1}, ..., l_{q,Nq}, l_5, l_q\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|cccccc} & l_{q,1} & l_{q,2} & ... & l_{q,Nq} & l_5 & l_q \\ \hline l_2 & W_{2,q1} & W_{2,q2} & ... & W_{2,qNq} & true & W_{2,q} \\ l_q & false & false & ... & false & false & true \end{array}$$

and

- $W_{2,qi}$ ="The output place $l_{q,i}$ is the highest priority empty place among the places $l_{q,1}, l_{q,2}, ..., l_{q,Nq}$", for $i = 1, 2, ..., Nq$;
- $W_{2,q} = W_{2,q1} \vee W_{2,q2} \vee ... \vee W_{2,qNq}$.

When the truth value of one of the predicates $W_{2,qi}$ for $i = 1, 2, ..., Nq$ is "true" the token of type α in place l_2 splits into three identical tokens: one of which enters the corresponding output place without obtaining new characteristic; the second merges with the γ token in place l_q and the third one enters place l_5 without obtaining new characteristics. The γ token in place l_q obtains the characteristic "*current value of* Y_q, T_q, F_q*; list of all tokens in the output places* $l_{q,1}, l_{q,2}, ..., l_{q,Nq}$ *and the duration of their stay in the corresponding place*".

$$Z_3 = \langle \{l_{q,1}, ..., l_{q,Nq}, l_s\}, \{l_6, l_s\}, r_3 \rangle,$$

where

$$r_3 = \begin{array}{c|cc} & l_6 & l_s \\ \hline l_{q,1} & W_{q1,6} & false \\ \vdots & \vdots & \vdots \\ l_{q,Nq} & W_{qNq,6} & false \\ l_s & false & true \end{array}$$

and

- $W_{qi,6}$ ="the current token has stayed more time in the place than the tokens in all other places $l_{q,1}, l_{q,2}, ..., l_{q,Nq}$ and $Y_s < N_s$" for $i = 1, 2, ..., Nq$.

When the truth value of one of the predicates $W_{qi,6}$ is "true" the token of type α from the corresponding place $l_{q,i}$ enters place l_6 without obtaining new characteristic. The δ token in place l_s obtains the characteristic "*current values of* Ys, Ts, Fs".

$$Z_4 = \langle \{l_5, l_6, l_{qs}\}, \{l_7, l_{qs}\}, r_4 \rangle,$$

where

$$r_4 = \begin{array}{c|cc} & l_7 & l_{qs} \\ \hline l_5 & false & true \\ l_6 & true & true \\ l_{qs} & false & true \end{array}.$$

The token of type α in place l_6 splits into two identical tokens one of which enters place l_7 without obtaining new characteristic. The other one merges with the ϵ token in place l_{qs}. The token of type α from place l_5 enters place l_{qs} where it merges with the ϵ token. The ϵ token in place l_{qs} obtains the characteristic *"current values of* $Y_{qs}, P_{qs}, F_{qs}, T_{qs}, N_{qs}$*"*.

$$Z_5 = \langle \{l_3, l_7, l_x\}, \{l_8, l_x\}, r_5 \rangle,$$

where

$$r_5 = \begin{array}{c|cc} & l_8 & l_x \\ \hline l_3 & false & true \\ l_7 & true & true \\ l_x & false & true \end{array}.$$

The token of type α in place l_7 splits into two identical tokens one of which enters place l_8 without obtaining new characteristic. The other one merges with the ζ token in place l_x. The token of type α from place l_3 enters place l_x where it merges with the ζ token. The ζ token in place l_x obtains the characteristic *"current values of* Y_x, P_x, F_x, T_x, N_x*"*.

IV. Detailed GN model of the causal structure

A more detailed representation of the causal structure of a queuing system is shown in Fig. 6. It takes into account two types of service of the requests in the buffer - without waitintg (base virtual causal device **zer.q**) and with waiting (base virtual causal device **gen.q**). The qualifiers used in the model are: gen. - comes from "genuine service"; zer.- comes from "zero service"; par. - comes from "parasitic"; nsr. - comes "not served". The comprise virtual devices are on three levels.

A GN representation of this model is shown in Fig. 7.

The GN consists of 9 transitions and $23 + Nq$ places, where Nq is the capacity of the buffer. The labels of those places which represent virtual devices are in the form l_y where "y" is the name of the corresponding virtual device but the "." symbol, if present in the name of the device, has been omitted. The transitions represent functions of the corresponding virtual devices as follows:

- Z_1 represents the function of the Director outgoing of the first Switch device in Fig. 6.

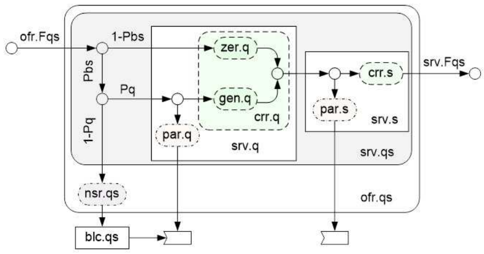

Fig. 6. A more detailed graphical representation of the causal structure of a queuing system in Service Systems Theory.

- Z_2 represents the function of the Director pointing to the **blc.qs** device from Fig. 6.
- Z_3 represents the function of the Switch before the **crr.q** causal device inside the **srv.q** causal device in Fig. 6.
- Z_4 represents the function of the Director entering the causal device **gen.q** in Fig. 6.
- Z_5 represents the function of the causal device **zer.q** from Fig. 6.
- Z_6 represents the function of the causal device **gen.q** in Fig.6.
- Z_7 represents the function of the Switch device to which the requests are sent from the causal devices **zer.q** and **gen.q** inside the causal device **crr.q** in Fig.6.
- Z_8 represents the function of the Switch device inside the **srv.s** device in Fig.6.
- Z_9 represents the function of the comprise devices **srv.qs** and **ofr.qs** in Fig.6.

Eleven types of tokens are used in the model:

- Tokens of type α represent the requests. In the initial time moment of the GN functioning, token α stays in place l_1 with initial characteristic "formula for generating the offered flow of requests to the queuing system".
- Token of type β stays in place l_{blcqs} in the initial time moment with initial characteristic "initial values of $Y_{blc.qs}, P_{blc.qs}, F_{blc.qs}, T_{blc.qs}, N_{blc.qs}$ ". It is used to accumulate data about the **blc.qs** device.
- Token of type γ stays in place l_{parq} in the initial time moment with initial characteristic "initial values of $Y_{par.q}, P_{par.q}, F_{par.q},\ T_{par.q}, N_{par.q}$". It is used to accumulate data about the **par.q** device. The discipline of service of the requests can be specified in this initial characteristic. Here, we consider only FIFO discipline of service of the requests.
- Token of type δ stays in place l_{zerq} in the initial time moment with initial characteristic "initial values of $Y_{zer.q}, P_{zer.}, F_{zer.q}, T_{zer.q}, N_{zer.q}$". It is used to accumulate data about the **zer.q** device.
- Token of type ϵ stays in place l_{genq} in the initial time moment with initial characteristic "initial values of $Y_{gen.q}, P_{gen.q}, F_{gen.q},\ T_{gen.q}, N_{gen.q}$". It is used to accumulate data about the **gen.q** device.
- Token of type ζ stays in place l_{srvq} in the initial time moment with initial characteristic "initial values of $Y_{srv.q}, P_{srv.q}, F_{srv.q},\ T_{srv.q}, N_{srv.q}$". It is used to accumulate data about the **srv.q** device.
- Token of type η stays in place l_{crrq} in the initial time moment with initial characteristic "initial values of $Y_{crr.q}, P_{crr.q}, F_{crr.q},\ T_{crr.q}, N_{crr.q}$". It is used to accumulate data about the **crr.q** device.
- Token of type θ stays in place l_{pars} in the initial time moment with initial characteristic "initial values of $Y_{par.s}, P_{par.s}, F_{par.s},\ T_{par.s}, N_{par.s}$". It is used to accumulate data about the **par.s** device.
- Token of type κ stays in place l_{crrs} in the initial time moment with initial characteristic "initial values of $Y_{crr.s}, P_{crr.s}, F_{crr.s},\ T_{crr.s}, N_{crr.s}$". It is used to accumulate data about the **crr.s** device.
- Token of type λ stays in place l_{ofrqs} in the initial time moment with initial characteristic "initial values of $Y_{ofr.qs}, P_{ofr.qs}, F_{ofr.qs},\ T_{ofr.qs}, N_{ofr.qs}$". It is used to accumulate data about the **ofr.qs** device.
- Token of type μ stays in place l_{srvqs} in the initial time moment with initial characteristic "initial values of $Y_{srv.qs}, P_{srv.qs}, F_{srv.qs},\ T_{srv.qs}, N_{srv.qs}$". It is used to accumulate data about the **srv.qs** device.

The formal description of the GN transitions follows below.

$$Z_1 = \langle \{l_1\}, \{l_2, l_3\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|cc} & l_2 & l_3 \\ \hline l_1 & W_{1,2} & W_{1,3} \end{array}$$

and

- $W_{1,2}$ ="$Ysrv.s < Nsrv.s$";
- $W_{1,3} = \neg W_{1,2}$.

When the truth value of the predicate $W_{1,2}$ is "true" the α token from place l_1 enters place l_2 without obtaining new characteristic, i.e. the request is serviced without delay. When the truth value of the predicate $W_{1,3}$ is "true" the α token from place l_1 enters place l_3 without obtaining new characteristic.

$$Z_2 = \langle \{l_3, l_{blcqs}\}, \{l_4, l_5, l_{blcqs}\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|ccc} & l_4 & l_5 & l_{blcqs} \\ \hline l_3 & W_{3,4} & false & W_{3,blcqs} \\ l_{blcqs} & false & true & true \end{array}$$

and

- $W_{3,4}$ ="$Ysrv.q < Nsrv.q$";
- $W_{3,blcqs} = \neg W_{3,4}$.

When the truth value of the predicate $W_{3,4}$ is "true" the α token in place l_3 enters place l_4 without obtaining new characteristic. When the truth value of the predicate $W_{3,blcqs}$ is "true" the α token in place l_3 enters place l_{blcqs} where it merges with the β token. Token β in place l_{blcqs} splits into two tokens one of which enters place l_5 without obtaining

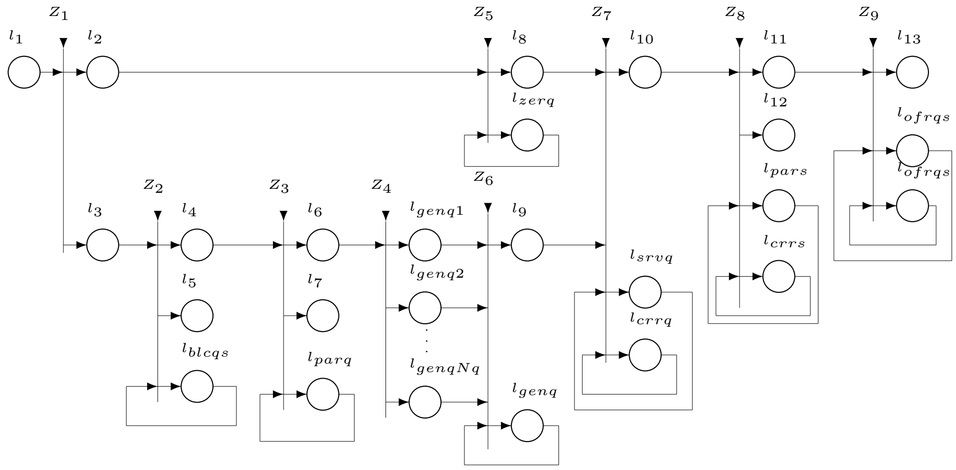

Fig. 7. GN model of a detailed representation of the causal structure of a queuing system.

new characteristc. The β token in place l_{blcqs} obtains the characteristic *"current values of $Y_{blc.qs}$"*.

$$Z_3 = \langle \{l_4, l_{parq}\}, \{l_6, l_7, l_{parq}\}, r_3 \rangle,$$

where

$$r_3 = \begin{array}{c|ccc} & l_6 & l_7 & l_{parq} \\ \hline l_4 & W_{4,6} & false & W_{4,parq} \\ l_{parq} & false & true & true \end{array}$$

and

- $W_{4,6}$ ="$Ygen.q < Ngen.q$";
- $W_{4,parq}$ ="the current request becomes parasitic (with given probability)".

When the truth value of the predicate $W_{4,6}$ is "true" α token in place l_4 enters place l_6 without obtaining new characteristic. When the truth value of the predicate $W_{4,parq}$ is "true" the α token in place l_4 enters place l_{parq} where it merges with the γ token. Token γ in place l_{parq} splits into two tokens one of which enters place l_7 without obtaining new characteristc. The γ token in place l_{parq} obtains the characteristic *"current values of $Y_{blc.qs}$"*.

$$Z_4 = \langle \{l_6\}, \{l_{genq,1}, ..., l_{genq,Nq}\}, r_4 \rangle,$$

where

$$r_4 = \begin{array}{c|cccc} & l_{genq,1} & l_{genq,2} & \cdots & l_{genq,Nq} \\ \hline l_6 & W_{6,genq1} & W_{6,genq2} & \cdots & W_{6,genqNq} \end{array}$$

and

- $W_{6,genqi}$ ="The output place $l_{genq,i}$ is the highest priority empty place among the places $l_{genq,1}, l_{genq,2}, ...,$ $l_{genq,Nq}$", for $i = 1, 2, ..., Nq$.

When the truth value of one of the predicates $W_{genq,i}$ for $i = 1, 2, ..., Nq$ is "true" the token of type α in place l_6 enters the corresponding output places where it obtains the characteristic *"time moment of entering the buffer"*.

$$Z_5 = \langle \{l_2, l_{zerq}\}, \{l_8, l_{zerq}\}, r_5 \rangle,$$

where

$$r_5 = \begin{array}{c|cc} & l_8 & l_{zerq} \\ \hline l_2 & true & false \\ l_{zerq} & false & true \end{array}.$$

The α token from place l_2 enters place l_8 without obtaining new characteristic. Token δ in place l_{zerq} obtains the characteristc *"current value of $Yzer.q$"*.

$$Z_6 = \langle \{l_{genq,1}, ..., l_{genq,Nq}, l_{genq}\}, \{l_9, l_{genq}\}, r_6 \rangle,$$

where

$$r_6 = \begin{array}{c|cc} & l_9 & l_{genq} \\ \hline l_{genq,1} & W_{genq1,9} & false \\ \vdots & \vdots & \vdots \\ l_{genq,Nq} & W_{genqNq,9} & false \\ l_{genq} & false & true \end{array}$$

and

- $W_{genqi,9}$ ="the current token has stayed more time in the place than the tokens in all other places $l_{genq,1}$, $l_{genq,2}, ..., l_{genq,Nq}$ and $Y_{srv.s} < N_{srv.s}$" for $i = 1, 2, ..., Nq$;

When the truth value of one of the predicates $W_{genqi,9}$ for $i = 1, 2, ..., Nq$ is "true" the α token in place $l_{genq,i}$ enters place l_9 without obtaining new characteristic. Token ϵ in place l_{genq} obtains the characteristic *"current value of $Ygen.q$"*.

$$Z_7 = \langle \{l_8, l_9, l_{srvq}, l_{crrq}\}, \{l_{10}, l_{srvq}, l_{crrq}\}, r_7 \rangle,$$

where

$$r_7 = \begin{array}{c|ccc} & l_{10} & l_{srvq} & l_{crrq} \\ \hline l_8 & true & false & false \\ l_9 & true & false & false \\ l_{srvq} & false & true & false \\ l_{crrq} & false & false & true \end{array}.$$

The tokens from places l_8 and l_9 merge in place l_{10} without obtaining new characteristic. Token ζ in l_{srvq} obtains the characteristic *"current value of* $Ysrv.q$*"*. Token η in l_{crrq} obtains the characteristic *"current value of* $Ycrr.q$*"*.

$$Z_8 = \langle \{l_{10}, l_{pars}, l_{crrs}\}, \{l_{11}, l_{12}, l_{pars}, l_{crrs}\}, r_8 \rangle,$$

where

$$r_8 = \begin{array}{c|cccc} & l_{11} & l_{12} & l_{pars} & l_{crrs} \\ \hline l_{10} & false & false & W_{10,pars} & W_{10,crrs} \\ l_{pars} & false & true & true & false \\ l_{crrs} & true & false & false & true \end{array}$$

and

- $W_{10,pars}$ = "the current request is parasitic (with given probability)";
- $W_{10,crrs}$ = "the current request is carried (with given probability)".

When the truth value of the predicate $W_{10,pars}$ is "true" the α token from place l_{10} enters place l_{pars} where it merges with token θ. Token θ in l_{pars} splits into two tokens one of which enters place l_{12} without obtaining new characteristic. In place l_{pars} token θ obtains the characteristic *"current value of* $Ypar.s$*"*. When the truth value of the predicate $W_{10,crrs}$ is "true" the α token from place l_{10} enters place l_{crrs} where it merges with token κ. Token κ in l_{crrs} splits into two tokens one of which enters place l_{11} without obtaining new characteristic. In place l_{crrs} token κ obtains the characteristic *"current value of* $Ycrr.s$*"*.

$$Z_9 = \langle \{l_{11}, l_{ofrqs}, l_{srvqs}\}, \{l_{13}, l_{ofrqs}, l_{srvqs}\}, r_9 \rangle,$$

where

$$r_9 = \begin{array}{c|ccc} & l_{13} & l_{ofrqs} & l_{srvqs} \\ \hline l_{11} & true & false & false \\ l_{ofrqs} & false & true & false \\ l_{srvqs} & false & false & true \end{array}.$$

Token α from place l_{11} enters place l_{13} without obtaining new characteristic. In place l_{ofrqs} token λ obtains the characteristic *"current value of* $Yofr.qs$*"*. In place l_{srvqs} token μ obtains the characteristic *"current value of* $Ysrv.qs$*"*.

V. Conclusion

In the present paper, for the first time the causal structure of a queuing system containing a buffer and a server, is modeled through GNs. A GN representation of comprise virtual devices is proposed which can be used in the conceptual models of overall telecommunication systems. The two GN models of the causal structure described here can be used in the study of the service quality composition of queuing systems in telecommunication networks.

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References

[1] M. Varela, P. Zwickl, P. Reichl, M. Xie, H. Schulzrinne. From Service Level Agreements (SLA) to Experience Level Agreements (ELA): The Challenges of Selling QoE to the User.*IEEE ICCW 2015*, 1741-1746, 2015.

[2] ITU-T Recommendation ITU-T P.10/G.100 (11/2017). *Vocabulary for performance, quality of service and quality of experience.*

[3] ITU-T Recommendation E.800, Definitions of Terms Related to Quality of Service, *ITUT Std.*, 2008.

[4] S. Poryazov and E. Saranova, *Models of Telecommunication Networks with Virtual Channel Switching and Applications*, Prof. Marin Drinov, Academic Publishing House, 2012, pp. 238 (In Bulgarian).

[5] S. Poryazov, E. Saranova, I. Ganchev, Scalable Traffic Quality and System Efficiency Indicators Towards Overall Telecommunication Systems QoE Management, *In: Ganchev I., van der Mei R., van den Berg H. (eds) Autonomous Control for a Reliable Internet of Services. State-of-the-Art Survey, Lecture Notes in Computer Science*, vol. 10768. Springer, Cham, 81-103, 2018.

[6] V. Andonov, S. Poryazov, A. Otsetova, E. Saranova, A Queue in Overall Telecommunication System with Quality of Service Guarantees. In: Poulkov V. (eds) Future Access Enablers for Ubiquitous and Intelligent Infrastructures. FABULOUS 2019. Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering, vol 283. Springer, Cham, 2019, 243-262.

[7] Boudewijn R. Haverkort, Performance of Computer Communication Systems: A Model-Based Approach. New York: John Wiley & Sons, 1998.

[8] K. Atanassov, *On Generalized Nets Theory*. Prof. M. Drinov Academic Publ. House, Sofia, Bulgaria, 2007.

[9] S. Poryazov, K. Atanassov, Generalized Net Subscribers' Traffic Model of Communication Switching Systems. *Journal on Advances in Modelling & Analysis*, Vol. 37, 1997, No. 1-2, 27-35.

[10] Zh. Tomov, M. Krawczak, V. Andonov, E. Dimitrov, K. Atanassov, Generalized net models of queueing disciplines in finite buffer queueing systems. *Proceedings of the 16th International Workshop on Generalized Nets*, 10 Feb. 2018, Sofia, 1-9.

[11] Zh. Tomov, M. Krawczak, V. Andonov, K. Atanassov, S. Simeonov, Generalized net models of queueing disciplines in finite buffer queueing systems with intuitionistic fuzzy evaluations of the tasks. *Notes on Intuitionistic Fuzzy Sets*, Vol 25, 2019, No 2, 115-122.

[12] S. Poryazov, V. Andonov, E. Saranova, Comparison of Four Conceptual Models of a Queuing System in Service Networks. Proc. of the 26th National conference with international participation TELECOM 2018, Sofia, 25-26 October, 71-77, 2018.

[13] V. Andonov, S. Poryazov, E. Saranova, Generralized net representations of elements of service systems theory. Advanced studies in contemporary mathematics, 29, 2019, No 2, 179-189.